DPP - Daily Practice Problems

Name :

Date :

Start Time :

End Time :

PHYSICS 27

SYLLABUS : Oscillations-1 (Periodic motion - period, Frequency, Displacement as a function of time. Periodic functions, Simple harmonic motion and its equation, Energy in S.H.M. - kinetic and potential energies)

Max. Marks : 120 **Time : 60 min.**

GENERAL INSTRUCTIONS

- The Daily Practice Problem Sheet contains 30 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.
- You have to evaluate your Response Grids yourself with the help of solution booklet.
- Each correct answer will get you 4 marks and 1 mark shall be deduced for each incorrect answer. No mark will be given/ deducted if no bubble is filled. Keep a timer in front of you and stop immediately at the end of 60 min.
- The sheet follows a particular syllabus. Do not attempt the sheet before you have completed your preparation for that syllabus. Refer syllabus sheet in the starting of the book for the syllabus of all the DPP sheets.
- After completing the sheet check your answers with the solution booklet and complete the Result Grid. Finally spend time to analyse your performance and revise the areas which emerge out as weak in your evaluation.

DIRECTIONS (Q.1-Q.22) : There are 22 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** choice is correct.

Q.1 A simple harmonic motion is represented by $F(t) = 10\sin(20t + 0.5)$. The amplitude of the S.H.M. is

(a) $a = 30$ cm (b) $a = 20$ cm
(c) $a = 10$ cm (d) $a = 5$ cm

Q.2 A particle executes a simple harmonic motion of time period T. Find the time taken by the particle to go directly from its mean position to half the amplitude

(a) $T/2$ (b) $T/4$ (c) $T/8$ (d) $T/12$

Q.3 The periodic time of a body executing simple harmonic motion is 3 sec. After how much time from time $t = 0$, its displacement will be half of its amplitude

(a) $\frac{1}{8}$sec (b) $\frac{1}{6}$sec (c) $\frac{1}{4}$sec (d) $\frac{1}{3}$sec

Q.4 If $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x' = a\cos\omega t$, then what is the phase difference between the two waves?

(a) $\pi/3$ (b) $\pi/6$
(c) $\pi/2$ (d) π

Q.5 A body is executing S.H.M. when its displacement from the mean position is 4 cm and 5 cm, the corresponding velocity of the body is 10 cm/sec and 8 cm/sec. Then the time period of the body is

(a) 2π sec (b) $\pi/2$ sec
(c) π sec (d) $3\pi/2$ sec

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ

Space for Rough Work

Q.6 If a simple pendulum oscillates with an amplitude of 50 mm and time period of 2 sec, then its maximum velocity is

(a) 0.10 m/s (b) 0.15 m/s
(c) 0.8 m/s (d) 0.26 m/s

Q.7 The maximum velocity and the maximum acceleration of a body moving in a simple harmonic oscillator are 2 m/s and 4 m/s^2. Then angular velocity will be

(a) 3 rad/sec (b) 0.5 rad/sec
(c) 1 rad/sec (d) 2 rad/sec

Q.8 The amplitude of a particle executing SHM is 4 cm. At the mean position the speed of the particle is 16 cm/sec. The distance of the particle from the mean position at which the speed of the particle becomes $8\sqrt{3}$ cm/s, will be

(a) $2\sqrt{3}$ cm (b) $\sqrt{3}$ cm
(c) 1 cm (d) 2 cm

Q.9 The amplitude of a particle executing S.H.M. with frequency of 60 Hz is 0.01 m. The maximum value of the acceleration of the particle is

(a) $144\pi^2 \text{m/sec}^2$ (b) 144 m/sec^2
(c) $\frac{144}{\pi^2} \text{m/sec}^2$ (d) $288\pi^2 \text{m/sec}^2$

Q.10 A particle executes simple harmonic motion with an angular velocity and maximum acceleration of 3.5rad/sec and 7.5 m/s^2 respectively. The amplitude of oscillation is

(a) 0.28 m (b) 0.36 m (c) 0.53 m (d) 0.61 m

Q.11 What is the maximum acceleration of the particle doing the SHM $y = 2\sin\left[\frac{\pi t}{2} + \phi\right]$ where y is in cm?

(a) $\frac{\pi}{2} \text{cm/s}^2$ (b) $\frac{\pi^2}{2} \text{cm/s}^2$
(c) $\frac{\pi}{4} \text{cm/s}^2$ (d) $\frac{\pi}{4} \text{cm/s}^2$

Q.12 The total energy of a particle executing S.H.M. is proportional to

(a) Displacement from equilibrium position
(b) Frequency of oscillation
(c) Velocity in equilibrium position
(d) Square of amplitude of motion

Q.13 When the displacement is half the amplitude, the ratio of potential energy to the total energy is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) $\frac{1}{8}$

Q.14 A particle is executing simple harmonic motion with frequency f. The frequency at which its kinetic energy changes into potential energy is

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

Q.15 A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

Q.16 The kinetic energy of a particle executing S.H.M. is 16 J when it is in its mean position. If the amplitude of oscillations is 25 cm and the mass of the particle is 5.12 kg, the time period of its oscillation is

(a) $\frac{\pi}{5}$ sec (b) 2π sec (c) 20π sec (d) 5π sec

Q.17 The displacement x (in metres) of a particle performing simple harmonic motion is related to time t (in seconds) as $x = 0.05\cos\left(4\pi t + \frac{\pi}{4}\right)$. The frequency of the motion will be

(a) 0.5 Hz (b) 1.0 Hz (c) 1.5 Hz (d) 2.0 Hz

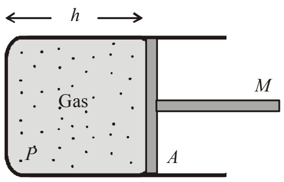
Q.18 A particle executes simple harmonic motion [amplitude = A] between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then

(a) $T_1 < T_2$ (b) $T_1 > T_2$ (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

RESPONSE GRID					
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ
	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ		

Space for Rough Work

Q.19 A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be



(a) $T = 2\pi\sqrt{\left(\frac{Mh}{PA}\right)}$ (b) $T = 2\pi\sqrt{\left(\frac{MA}{Ph}\right)}$

(c) $T = 2\pi\sqrt{\left(\frac{M}{PAh}\right)}$ (d) $T = 2\pi\sqrt{MPhA}$

Q.20 A particle is performing simple harmonic motion along x-axis with amplitude 4 cm and time period 1.2 sec. The minimum time taken by the particle to move from $x = 2$ cm to $x = +4$ cm and back again is given by

(a) 0.6 sec (b) 0.4 sec
(c) 0.3 sec (d) 0.2 sec

Q.21 A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of

(a) (2/3)k (b) (3/2)k
(c) 3k (d) 6k

Q.22 A simple pendulum has time period T_1. The point of suspension is now moved upward according to equation $y = kt^2$ where $k = 1 m/sec^2$. If new time period is T_2 then ratio $\frac{T_1^2}{T_2^2}$ will be

(a) 2/3 (b) 5/6
(c) 6/5 (d) 3/2

DIRECTIONS (Q.23-Q.25) : In the following questions, more than one of the answers given are correct. Select the correct answers and mark it according to the following codes:

Codes : **(a)** 1, 2 and 3 are correct
(b) 1 and 2 are correct
(c) 2 and 4 are correct
(d) 1 and 3 are correct

Q.23 A particle constrained to move along the x-axis in a potential $V = kx^2$, is subjected to an external time dependent force $\vec{f}(t)$, here k is a constant, x the distance from the origin, and t the time. At some time T, when the particle has zero velocity at x = 0, the external force is removed. Choose the incorrect options –

(1) Particle executes SHM
(2) Particle moves along +x direction
(3) Particle moves along – x direction
(4) Particle remains at rest

Q.24 Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then –

(1) The resultant amplitude is $(1+\sqrt{2})\,a$
(2) The phase of the resultant motion relative to the first is 90°
(3) The energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion
(4) The resulting motion is not simple harmonic

Q.25 For a particle executing simple harmonic motion, which of the following statements is correct?

(1) The total energy of the particle always remains the same
(2) The restoring force always directed towards a fixed point
(3) The restoring force is maximum at the extreme positions
(4) The acceleration of the particle is maximum at the equilibrium position

RESPONSE GRID					
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	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ			

Space for Rough Work

DIRECTIONS (Q.26-Q.27) : Read the passage given below and answer the questions that follows :

The differential equation of a particle undergoing SHM is given by $a\frac{d^2x}{dt^2} + bx = 0$. The particle starts from the extreme position.

Q.26 The ratio of the maximum acceleration to the maximum velocity of the particle is –

(a) $\frac{b}{a}$ (b) $\frac{a}{b}$

(c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

Q.27 The equation of motion may be given by :

(a) $x = A\sin\left(\sqrt{\frac{b}{a}}\right)t$

(b) $x = A\cos\left(\sqrt{\frac{b}{a}}\right)t$

(c) $x = A\sin\left(\sqrt{\frac{b}{a}}t + \theta\right)$ where $\theta \neq \pi/2$

(d) None of these

DIRECTIONS (Q.28-Q.30) : Each of these questions contains two statements: Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement -1 is False, Statement-2 is True.

(d) Statement -1 is True, Statement-2 is False.

Q.28 Statement-1 : In S.H.M., the motion is 'to and fro' and periodic.

Statement-2 : Velocity of the particle $(v) = \omega\sqrt{k^2 - x^2}$ (where x is the displacement and k is amplitude)

Q.29 Statement-1 : In simple harmonic motion, the velocity is maximum when acceleration is minimum.

Statement-2 : Displacement and velocity of S.H.M. differ in phase by $\pi/2$

Q.30 Statement-1 : The graph of total energy of a particle in SHM *w.r.t.*, position is a straight line with zero slope.

Statement-2 : Total energy of particle in SHM remains constant throughout its motion.

RESPONSE GRID					
	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM SHEET 27 - PHYSICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	30	Qualifying Score	45
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Space for Rough Work

DAILY PRACTICE PROBLEMS

PHYSICS SOLUTIONS

27

1. (c) $a = 10$

2. (d) $y = A\sin\omega t = \frac{A\sin 2\pi}{T}t \Rightarrow \frac{A}{2} = A\sin\frac{2\pi t}{T} \Rightarrow t = \frac{T}{12}$

3. (c) $y = a\sin\frac{2\pi}{T}t \Rightarrow \frac{a}{2} = a\sin\frac{2\pi t}{3} \Rightarrow \frac{1}{2} = \sin\frac{2\pi t}{3}$

$\Rightarrow \sin\frac{2\pi t}{3} = \sin\frac{\pi}{6} \Rightarrow \frac{2\pi t}{3} = \frac{\pi}{6} \Rightarrow t = \frac{1}{4}\text{sec}$

4. (a) $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x' = a\cos\omega t = a\sin\left(\omega t + \frac{\pi}{2}\right)$

$\therefore \Delta\phi = \left(\omega t + \frac{\pi}{2}\right) - \left(\omega t + \frac{\pi}{6}\right) = \frac{\pi}{3}$

5. (c) $v = \omega\sqrt{a^2 - y^2} \Rightarrow 10 = \omega\sqrt{a^2 - (4)^2}$ and

$8 = \omega\sqrt{a^2 - (5)^2}$

On solving, $\omega = 2 \Rightarrow \omega = \frac{2\pi}{T} = 2 \Rightarrow T = \pi\,\text{sec}$

6. (b) $v_{max} = a\omega = a \times \frac{2\pi}{T} = (50\times 10^{-3}) \times \frac{2\pi}{2} = 0.15\, m/s$

7. (d) $v_{max} = a\omega$ and $A_{max} = a\omega^2$

$\Rightarrow \omega = \frac{A_{max}}{v_{max}} = \frac{4}{2} = 2\, rad/sec$

8. (d) At mean position velocity is maximum

i.e., $v_{max} = \omega a \Rightarrow \omega = \frac{v_{max}}{a} = \frac{16}{4} = 4$

$\therefore \upsilon = \omega\sqrt{a^2 - y^2} \Rightarrow 8\sqrt{3} = 4\sqrt{4^2 - y^2}$

$\Rightarrow 192 = 16(16 - y^2) \Rightarrow 12 = 16 - y^2 \Rightarrow y = 2cm$

9. (a) Maximum acceleration $= a\omega^2 = a \times 4\pi^2 n^2$

$= 0.01 \times 4 \times (\pi)^2 \times (60)^2 = 144\pi^2 m/\text{sec}$

10. (d) $A_{max} = a\omega^2 \Rightarrow a = \frac{A_{max}}{\omega^2} = \frac{7.5}{(3.5)^2} = 0.61\, m$

11. (b) Comparing given equation with standard equation,

$y = a\sin(\omega t + \phi)$, we get, $a = 2cm, \omega = \frac{\pi}{2}$

$\therefore A_{max} = \omega^2 A = \left(\frac{\pi}{2}\right)^2 \times 2 = \frac{\pi^2}{2} cm/s^2$

12. (d) $E = \frac{1}{2}m\omega^2 a^2 \Rightarrow E \propto a^2$

13. (b) $\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2} = \frac{\left(\frac{a}{2}\right)^2}{a} = \frac{1}{4}$

14. (c) In S.H.M., frequency of K.E. and P.E. $= 2 \times$ (Frequency of oscillating particle)

15. (c) Kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2\omega^2\cos^2\omega t$

$= \frac{1}{2}m\omega^2 a^2(1 + \cos 2\omega t)$

hence kinetic energy varies periodically with double the frequency of S.H.M. *i.e.* 2ω.

16. (a) At mean position, the kinetic energy is maximum.

Hence $\frac{1}{2}ma^2\omega^2 = 16$

On putting the values we get

$\omega = 10 \Rightarrow T = \frac{2\pi}{\omega} = \frac{\pi}{5}\text{sec}$

17. (d) From the given equation, $\omega = 2\pi n = 4\pi \Rightarrow n = 2Hz$

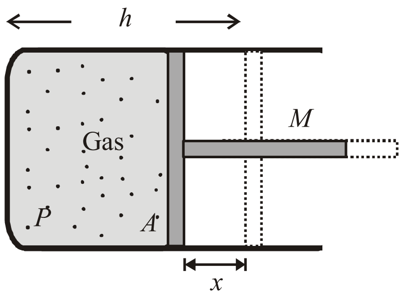
18. (a) Using $x = A\sin\omega t$

For $x = A/2$, $\sin\omega T_1 = 1/2 \Rightarrow T_1 = \frac{\pi}{6\omega}$

For $x = A$, $\sin\omega(T_1 + T_2) = 1 \Rightarrow T_1 + T_2 = \frac{\pi}{2\omega}$

$\Rightarrow T_2 = \frac{\pi}{2\omega} - T_1 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$ *i.e.*, $T_1 < T_2$

19. (a) Let the piston be displaced through distance x towards left, then volume decreases, pressure increases. If ΔP is increased in pressure and ΔV is decreased in volume, then considering the process to take place gradually (*i.e* isothermal)



$P_1V_1 = P_2V_2 \Rightarrow PV = (P + \Delta P)(V - \Delta V)$

$\Rightarrow PV = PV + \Delta PV - P\Delta V - \Delta P\Delta V$

$\Rightarrow \Delta P.V - P.\Delta V = 0$ (neglecting $\Delta P.\Delta V$)

$\Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \frac{P.x}{h}$

This excess pressure is responsible for providing the restoring force (F) to the piston of mass M.

Hence $F = \Delta P.A = \frac{PAx}{h}$

Comparing it with $|F| = kx \Rightarrow k = M\omega^2 = \frac{PA}{h}$

$\Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi\sqrt{\frac{Mh}{PA}}$

20. (b) Time taken by particle to move from $x = 0$ (mean position) to $x = 4$ (extreme position)

$= \frac{T}{4} = \frac{1.2}{4} = 0.3\,s$

Let t be the time taken by the particle to move from $x = 0$ to $x = 2\,cm$

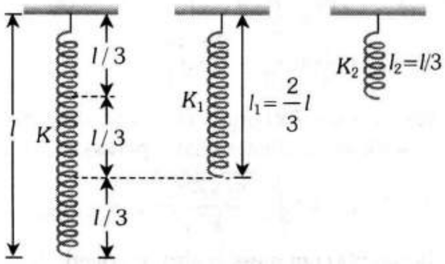
$y = a\sin\omega t \Rightarrow 2 = 4\sin\frac{2\pi}{T}t \Rightarrow \frac{1}{2} = \sin\frac{2\pi}{1.2}t$

$\Rightarrow \frac{\pi}{6} = \frac{2\pi}{1.2}t \Rightarrow t = 0.1s$.

Hence time to move from $x = 2$ to $x = 4$ will be equal to $0.3 - 0.1 = 0.2\,s$

Hence total time to move from $x = 2$ to $x = 4$ and back again $= 2 \times 0.2 = 0.4\,\text{sec}$

21. (b)



Force constant $(k) \propto \frac{1}{\text{Length of spring}}$

$\Rightarrow \frac{K}{K_1} = \frac{l_1}{l} = \frac{\frac{2}{3}l}{l} \Rightarrow K_1 = \frac{3}{2}K.$

22. (c) $y = Kt^2 \Rightarrow \frac{d^2y}{dt^2} = a_y = 2K = 2 \times 1 = 2\,m/s^2$ $(\because K = 1\,m/s^2)$

Now, $T_1 = 2\pi\sqrt{\frac{l}{g}}$ and $T_2 = 2\pi\sqrt{\frac{l}{(g + a_y)}}$

Dividing, $\frac{T_1}{T_2} = \sqrt{\frac{g + a_y}{g}} \Rightarrow \sqrt{\frac{6}{5}} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{6}{5}$

23. (a) At x = 0, v = 0 and potential energy is minimum so particle will remain at rest.

24. (d) Let simple harmonic motions be represented by

$y_1 = a\sin\left(\omega t - \frac{\pi}{4}\right)$, $y_2 = a\sin\omega t$

and $y_3 = a\sin\left(\omega t + \frac{\pi}{4}\right)$

On superimposing, resultant SHM will be

$y = a\left[\sin\left(\omega t - \frac{\pi}{4}\right) + \sin\omega t + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$

$= a\left[2\sin\omega t\cos\frac{\pi}{4} + \sin\omega t\right] = a[\sqrt{2}\sin\omega t + \sin\omega t]$

$= a(1 + \sqrt{2})\sin\omega t$

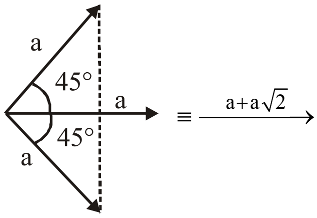
Resultant amplitude $= (1 + \sqrt{2})a$

Energy in SHM $\propto$ (Amplitude)^2

$\therefore \frac{E_{Resultant}}{E_{Single}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2} + 1)^2 = (3 + 2\sqrt{2})$

$\Rightarrow E_{resultant} = (3 + 2\sqrt{2})\,E_{single}$

OR



25. (a) Acceleration $\propto$ – displacement, and direction of acceleration is always directed towards the equilibrium position.

26. (d)

27. (b)

Compare given equation with $\frac{d^2x}{dt^2} + \omega^2 x = 0$; $\omega^2 = \frac{b}{a}$

$\frac{a_{max}}{v_{max}} = \frac{\omega^2 A}{\omega A} = \omega = \sqrt{\frac{b}{a}}$

At $t = 0$, $\phi = \pi/2$

$x = A\sin(\omega t + \phi) = A\cos\sqrt{\frac{b}{a}}t$

28. (b)

29. (b) $x = a\sin\omega t$ and $v = \frac{dx}{dt} = a\omega\cos\omega t$

It is clear that phase difference between 'x' and 'a' is $\frac{\pi}{2}$.

30. (a) The total energy of S.H.M. = Kinetic energy of particle + potential energy of particle.
The variation of total energy of the particle in SHM with time is shown in a graph.